Math 4740 - Homework # 1
Sets and Probability Spaces

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1. (a) Consider the experiment of rolling two 6-sided dice. What is a sample space S and a probability function P for this experiment? (b) Let A be the event that the sum of the dice is 2. Let B be the event that the sum of the dice is 4. What are the elements of A and B? (c) Calculate $A \cup B$, $A \cap B$, $\overline{A}$, $\overline{B}$. (d) How do the sets from part c relate to logic? What logical sentences express them? (e) Calculate the probabilities of the events from parts b and c.

2. Consider the experiment of rolling two 6-sided dice. (a) What is a sample space S and a probability function P for this experiment? (b) Let A be the event that at least one of the dice is even. Let B be the event that both of the dice are even. Let C be the event that at least one of the dice is odd. Let D be the event that both of the dice are odd. List the elements of A, B, C, and D. (c) Calculate $A \cap C$, $\overline{A}$, $B \cap D$, $B \cup D$, $\overline{B}$, $\overline{D}$. (d) How do the sets from part c relate to logic? What logical sentences express them? (e) Calculate the probabilities of the events in parts b and c.

3. Consider the experiment of flipping a coin 4 times in a row. (a) What is a sample space S and a probability function P for this experiment? (b) Let A be the event that a head occurs on the first flip. Write out the elements of A. Let B be the event that a tails occurs on the second flip and the fourth flip. Write out the elements of B. (c) Calculate $A \cup B$, $A \cap B$, $\overline{A}$, $\overline{B}$. (d) How do these sets relate to logic? What logical sentences express them? (e) Calculate the probabilities of the events in parts b and c.

4. Consider the experiment of rolling a weighted 4-sided dice. Weighted means that the probabilities aren't equal for each number on the dice. Suppose that through experimentation you discover that the 1 occurs two times for every eight rolls of the dice, a 2 occurs two times for every eight rolls of the dice, a 3 occurs three times for every eight rolls of the dice, and a 4 occurs one time for every eight rolls of the dice. (a) What

is a sample space S and a probability function P for this experiment? (b) Let A be the event of rolling a 1 or a 3. What are the elements of A and what is the probability of A? (c) Let B be the event of rolling a 1 or a 2 or a 3. What are the elements of B and what is the probability of B?

5. Suppose that two 4-sided dice are thrown.
   (a) What is the probability that at least one of the dice shows a 2?
   (b) What is the probability that the sum of the dice is 4?
   (c) What is the probability that the sum of the dice is either a 5 or a 7?
6. Suppose that two 8-sided dice are thrown, one green and one red. What is the probability that the red die has a larger value than the green die?
7. Consider an bag that contains three balls: 1 white balls, 1 red ball, and 1 green ball. Suppose you want to construct a probabilistic model of the experiment where you randomly (without looking) choose 1 ball from the bag and record the result. How could you do such a thing? What is the sample space S? Describe the probability function P. What is the probability of choosing a white ball? Of choosing a red ball?
8. Suppose we take the same setup as problem 7. But we change it to the experiment where you randomly choose 2 balls from the bag. What is a possible sample space S and probability function P for this model? What is the probability of choosing one white ball and one red ball?

The next two problems 9 and 10 are optional. They involve infinite probability spaces.

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9. Consider the experiment where one 4-sided dice is thrown continually until a 2 is rolled. Construct a probability space that models this experiment. Verify that you have a probability space.
10. Consider the experiment where a coin is flipped continually until a head occurs. Construct a probability space that models this experiment. Verify that you have a probability space.